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The new method for finding adjoint matrix by eliminating ancient tedious method

Ruchi Bavda
Department of Mathematics, Maharaja Krishna Kumar Sinhji Bhavnagar University, Bhavnagar, Gujarat, India

Abstract
In this paper work I study the derivation of the adjoint of non-singular 3×3 matrices by uniquely eliminating the lengthy cofactor derivation method. Some computational and results were carried out to understand the concept.

Keywords: matrix, non-singular, adjoint matrix, Adjugate, advance method

1. Introduction
The Matrices are a rectangular array of numbers, symbols, or expressions arranged in row and column [1]. Matrices have a long history of application in solving linear equations. In this paper we are discussing about adjoint matrices. The Adjoint matrices of 3×3 non-singular matrix is derived with the help of cofactors. Adjoint of a matrix is defined as a transpose of the cofactor matrix of that particular matrix. For a matrix A, the adjoint is denoted as adj (A) [2, 3, 4].

1.1 Definitions
1.1.1 Matrix
A matrix is a collection of numbers arranged into a fixed number of rows (m) and columns (n). The individual quantities are called the elements of the matrix of rows and column is said to be of order m × n.
Equation-(1)

1.1.2 Determinant Matrix
The determinant of a matrix is a scalar (constant) obtained from the matrix by an appropriate evaluation depending on the order of the matrix. The determinant of matrix A is denoted by |A|.

1.1.3 Non-singular Matrix
This is a square matrix whose determinant is not zero.

1.1.4 Adjoint Matrix
This is the transpose of the co-factors matrix. The adjoint of a matrix A is denoted by (AdjA).

1.1.5 Transpose of matrix
In linear algebra, the transpose of a matrix is an operator which flips a matrix over its diagonal, that is it switches the row and column indices of the matrix by producing another matrix denoted as A^T.

2. Materials and methods
For general form of 3 × 3 matrix.
Equation-(2)
If we want to derive adjoint of matrix we have to find cofactor of given matrix. So, for cofactor we take Equation-(2) and write first two sequential column outside of the matrix gives,
Equation-(3)
By deleting first row and first column along with newly added column we get first row of cofactor as a result,
Equation-(3.1)
Equation- (3.1.1)
Now by adding a column into Equation-(3) below the matrix we get,
Equation-(3.2)
Deleting first and second row as well as first column we get second row of cofactor as a result,
Equation-(3.2.1)
Equation-(3.2.2)
For derivation of third row of cofactor add second row in Equation-(3.2) for result,
Equation- (3.3)
By deleting first second and third row along with first column we get third row of cofactor,
Equation-(3.3.1)
Equation- (3.3.2)
After putting all the obtained values of individual rows of cofactors in one matrix we get cofactor as,
Equation-(3.4)
For derivation of Adj-A,
Equation-(3.5)
As a result, we get Equation-(3.6) after transpose,
Equation-(3.6)

2.2. Examples
2.2.1 Equation-(4) find adjoint of a matrix.
As cofactor is basic requirement for deriving Adj(A), add first two sequential column adjacent to the matrix equation–(4) and we get,
Equation-(4.1)
We get first row of cofactor,
Equation-(4.1.1)
Equation-(4.1.2)
Equation-(4.1.3)
For second row of the cofactor add first row just below equation 4.1 and delete first and second row as well as first column along with newly added row,
Equation-(4.2)
Equation-(4.2.1)
Equation-(4.2.2)
Equation-(4.2.3)

Similarly, for third row of cofactor add second row in equation-(4.2) and we get,
Equation-(4.3)
By deleting first, second, third row and first column we get,
Equation-(4.3.1)
Equation-(4.3.2)
Equation-(4.3.3)
Equation-(4.4)
Equation-(4.5)
Equation-(4.6)

2.2.1 Equation-(5) find the adjoint of B.
For cofactor of B,
Equation-(5.1)
Equation-(5.2)
Equation-(5.3)
Equation-(5.4)
Equation-(5.5)
Equation-(5.6)

3. Tables and Figures

Table 1: Some example of matrix problem and its resultant Adjoint derivation by new proposed method.

Matrix		Adjoint
1.	$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 5 & 6 & 0 \end{bmatrix}$	$\begin{bmatrix} -30 & 18 & 7 \\ 25 & -15 & -5 \\ -5 & 4 & 1 \end{bmatrix}$
2	$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$
3	$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -2 & 0 & -2 \\ 2 & 0 & -4 \\ -1 & -3 & 2 \end{bmatrix}$
4	$\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -1 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 4 & 6 \\ -6 & 3 & -6 \\ -4 & -5 & 3 \end{bmatrix}$
5	$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 2 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$

4. Equations

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad (1)$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (2)$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix} \quad (3)$$

$$= \left[\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \quad \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} \quad \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \right] \quad (3.1)$$

$$[a_{22}a_{33} - a_{23}a_{32} \quad a_{23}a_{31} - a_{21}a_{33} \quad a_{21}a_{32} - a_{22}a_{31}] \quad (3.1.1)$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$$

$$a_{11} \quad a_{12} \quad a_{13} \quad a_{11} \quad a_{12} \quad (3.2)$$

$$= \left[\begin{vmatrix} a_{32} & a_{33} \\ a_{12} & a_{13} \end{vmatrix} \quad \begin{vmatrix} a_{33} & a_{31} \\ a_{13} & a_{11} \end{vmatrix} \quad \begin{vmatrix} a_{31} & a_{32} \\ a_{11} & a_{12} \end{vmatrix} \right] \quad (3.2.1)$$

$$[a_{32}a_{13} - a_{33}a_{12} \quad a_{33}a_{11} - a_{31}a_{13} \quad a_{31}a_{12} - a_{32}a_{11}] \quad (3.2.2)$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$$

$$a_{11} \quad a_{12} \quad a_{13} \quad a_{11} \quad a_{12}$$

$$a_{21} \quad a_{22} \quad a_{23} \quad a_{21} \quad a_{22} \qquad (3.3)$$

$$= \left[\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \quad \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \quad \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \right] \qquad (3.3.1)$$

$$[a_{32}a_{13} - a_{33}a_{12} \quad a_{33}a_{11} - a_{31}a_{13} \quad a_{31}a_{12} - a_{32}a_{11}] \qquad (3.3.2)$$

Co-factor of A

$$= \begin{bmatrix} a_{22}a_{33} - a_{23}a_{32} & a_{23}a_{31} - a_{21}a_{33} & a_{21}a_{32} - a_{22}a_{31} \\ a_{32}a_{13} - a_{33}a_{12} & a_{33}a_{11} - a_{31}a_{13} & a_{31}a_{12} - a_{32}a_{11} \\ a_{32}a_{13} - a_{33}a_{12} & a_{33}a_{11} - a_{31}a_{13} & a_{31}a_{12} - a_{32}a_{11} \end{bmatrix} \qquad (3.4)$$

$$\text{Adj A} = [co - factor]^T \qquad (3.5)$$

Adj A

$$= \begin{bmatrix} a_{22}a_{33} - a_{23}a_{32} & a_{32}a_{13} - a_{33}a_{12} & a_{32}a_{13} - a_{33}a_{12} \\ a_{23}a_{31} - a_{21}a_{33} & a_{33}a_{11} - a_{31}a_{13} & a_{33}a_{11} - a_{31}a_{13} \\ a_{21}a_{32} - a_{22}a_{31} & a_{31}a_{12} - a_{32}a_{11} & a_{31}a_{12} - a_{32}a_{11} \end{bmatrix} \qquad (3.6)$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{bmatrix} \qquad (4)$$

$$A = \begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & 2 & 3 \\ \mathbf{1} & -1 & 1 \end{bmatrix} \begin{matrix} \mathbf{2} & \mathbf{1} \\ 1 & 2 \\ 1 & -1 \end{matrix} \qquad (4.1)$$

$$= \left[\begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} \quad \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} \right] \qquad (4.1.1)$$

$$= [2 - (-3) \quad 3 - 1 \quad (-1) - 2] \qquad (4.1.2)$$

$$= [5 \quad 2 \quad -3] \qquad (4.1.3)$$

$$A = \begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{1} & -1 & 1 \end{bmatrix} \begin{matrix} \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} \\ 1 & -1 \end{matrix}$$

$$\mathbf{2} \quad 1 \quad 1 \quad 2 \quad 1 \qquad (4.2)$$

$$= \left[\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \quad \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} \right] \qquad (4.2.1)$$

$$= [(-1) - 1 \quad 2 - 1 \quad 1 - (-2)] \qquad (4.2.2)$$

$$= [-2 \quad 1 \quad 3] \qquad (4.2.3)$$

$$A = \begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{1} & \mathbf{-1} & \mathbf{1} \end{bmatrix} \begin{matrix} \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} \\ \mathbf{1} & \mathbf{-1} \end{matrix}$$

$$\mathbf{2} \quad 1 \quad 1 \quad 2 \quad 1$$

$$\mathbf{1} \quad 2 \quad 3 \quad 1 \quad 2 \qquad (4.3)$$

$$= \left[\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \quad \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} \quad \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \right] \qquad (4.3.1)$$

$$= [3 - 2 \quad 1 - 6 \quad 4 - 1] \qquad (4.3.2)$$

$$= [1 \quad -5 \quad 3] \qquad (4.3.3)$$

$$\text{Co – factor of A} = \begin{bmatrix} 5 & 2 & -3 \\ -2 & 1 & 3 \\ 1 & -5 & 3 \end{bmatrix} \quad (4.4)$$

$$\text{Adj A} = [co - factor]^T \quad (4.5)$$

$$\text{Adj A} = \begin{bmatrix} 5 & -2 & 1 \\ 2 & 1 & -5 \\ -3 & 3 & 3 \end{bmatrix} \quad (4.6)$$

$$\boldsymbol{B} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{-1} & \mathbf{2} & \mathbf{5} \\ \mathbf{4} & \mathbf{3} & \mathbf{1} \end{bmatrix} \quad (5)$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 5 \\ 4 & 3 & 1 \end{bmatrix} \begin{matrix} 1 & 2 \\ -1 & 2 \\ 4 & 3 \end{matrix}$$

$$\begin{matrix} 1 & 2 & 3 & 1 & 2 \\ -1 & 2 & 5 & -1 & 2 \end{matrix} \quad (5.1)$$

$$\text{Co- factor of B} = \begin{bmatrix} \begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 5 & -1 \\ 1 & 4 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 4 & 3 \end{vmatrix} \\ \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 5 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} \end{bmatrix} \quad (5.2)$$

$$\text{Co- factor of B} = \begin{bmatrix} -13 & 21 & -11 \\ 7 & -11 & 5 \\ 4 & -8 & 4 \end{bmatrix} \quad (5.3)$$

$$\text{Adj B} = [co\ factor]^T \quad (5.4)$$

$$\text{Adj B} = \begin{bmatrix} -13 & 21 & -11 \\ 7 & -11 & 5 \\ 4 & -8 & 4 \end{bmatrix}^T \quad (5.5)$$

$$Adj\ B = \begin{bmatrix} -13 & 7 & 4 \\ 21 & -11 & -8 \\ -11 & 5 & 4 \end{bmatrix} \quad (5.6)$$

5. Conclusions

In conclusion, it is apparent from table-1 that designed new method for finding adjoint of matrix is precise, accurate and cushy. We can conclude from computational and result that this method gives efficient result in accordance to ancient tedious method, while finding adjoint of matrix proposed new method is abbreviated comparative to ancient method.

6. References

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